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Holographic Fermi surfaces in the six-dimensional (2, 0) theory

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We analyze the Fermi surface behavior of fermionic modes from a charged black hole background with two different charges arising in a truncated gauged supergravity theory in seven dimensions, which is dual to the six-dimensional (2, 0) theory. At zero temperature, we numerically solve equations for fermionic modes that do not couple to the gravitino. We find that each of the modes admits at least one Fermi surface. We find Fermi surfaces in both the Fermi-liquid and non-Fermi-liquid regimes.

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I. INTRODUCTION

Metals and other interacting fermion systems are successfully described by the Fermi-liquid theory, where fermions are dressed into weakly coupled long-lived quasiparticles near the Fermi surface. However, cuprate superconductors and heavy fermions have evidence of a Fermi surface, but fluctuations are unstable and have been called non-Fermi liquid. In order to understand such systems, gauge-gravity correspondence [1–3] turns out to be quite useful, where the theory of fermion systems is described by a gravitational system with charged, asymptotically anti–de Sitter (AdS) black hole geometries in one higher dimension, which are computationally easier to analyze.

A study of Fermi surfaces using holography first appeared in Refs. [4–6]. Sharp quasiparticlelike fermionic excitations at low energies with scaling behavior different from Fermi liquid were found. It was also found that, over a finite range of momentum, the spectral function shows log periodic behavior. Subsequently, low-energy behavior of the system in general dimensions with constant mass and gauge couplings was considered in Ref. [7]. They studied, in particular, the relation between the scaling exponent of the spectral function and the dimension of the operator in the IR conformal field theory, among other things. The effect of Pauli coupling was studied in Refs. [8,9], and for the strength beyond a critical value they found a gap in the density of states.

These studies postulate a convenient effective Lagrangian on the gravity side and analyze the outcomes. Another approach is to begin with a known string or supergravity model. One advantage of studies of a known string or supergravity model is that the dual theory is often known. Variations of the parameters within the theory also help to make an identification of states in the dual theory. In this approach, Fermi surfaces were studied for fermions realized on probe branes [10,11]. Subsequently, the fermionic response of gravitinos in minimal $\mathcal{N} = 2$ supergravity theories [12–14] was analyzed, but they did not find a Fermi surface.

Afterwards, systematic studies of five- and four-dimensional maximally gauged supergravity with variations of the chemical potential appear where dual theories are $N = 4$ super-Yang-Mills (SYM) in four dimensions and Aharony-Bergman-Jaeris-Maldacena (ABJM) theory in three dimensions, respectively [15–17]. They analyzed spin-$1/2$ fermions in supergravity, which do not couple with the gravitino, and found that fermions with higher charges are more likely to have Fermi surfaces over a range of chemical potentials. In general, these are in the non-Fermi regime, except a set of fermions approaches marginal Fermi liquid for a certain limiting value of the chemical potentials. A model with a single charge having vanishing entropy at zero temperature was also analyzed in Ref. [18], where they found that fermionic fluctuations are stable within a gap around the Fermi surface. Discussions of Fermi surfaces in a similar context also appeared in Refs. [19–21].

In the present work, we have considered the study of Fermi surfaces in a maximally gauged supergravity theory in seven dimensions. The dual theory is the (2, 0) conformal field theory in six dimensions, which is the world volume theory of M5 branes. This theory is interesting in its own right, as this is one of the three maximally superconformal field theories. The other two, namely, $N = 4$ SYM and ABJM theory, have been studied in Refs. [15,17,18]. This theory consists of a tensor multiplet in six dimensions, and in that respect the field content is different. In particular, the gaugino appears as symplectic Majorana-Weyl spinors. So from such a study we expect to learn something new about this theory, and this will improve our understanding of the Fermi surface behavior, in general.

The seven-dimensional gauged supergravity has an $SO(5)$ R-symmetry group. We consider a black hole background with two chemical potentials at zero temperature giving rise to a one-parameter family. The supergravity contains 16 spin-$1/2$ fermions, of which we have studied only those modes that do not couple to the gravitino. We find that each mode admits a Fermi surface

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over a range of parameters, unless they vanish inside an oscillatory region where the Green function displayed log oscillatory behavior. We also find excitations in Fermi liquid regime and in non-Fermi liquid regime. In some cases, it approaches marginal Fermi-liquid behaviour near the boundary of the range.

The plan of the paper is as follows. In the next section, we discuss the charged black hole solution. Sections III and IV are devoted to fermionic fluctuations and the solution of the Dirac equations, respectively. Section V discusses the results.

II. BOSONIC ACTION

The M theory compactified on $AdS_7 \times S^4$ is conjectured to be dual to a conformal field theory in six dimensions. There exists a consistent truncation of 11-dimensional supergravity to the maximal ($N = 4$) seven-dimensional gauged supergravity with only the lowest massless modes [22], which in the appropriate limit can serve as a dual to the six-dimensional conformal field theory [1–3].

The bosonic content of the supergravity theory is as follows [22–24]. It has a gauged $SO(5)_g$ and a composite $SO(5)_c$ group. It contains a graviton, Yang-Mills gauge fields transforming under adjoint of gauge group $SO(5)_g$, five rank-3 tensor fields transforming as a 5 under $SO(5)_g$, and 14 scalars parametrizing the $SL(5,R)/SO(5)_c$ coset. The fermionic contents are four gravitini and 16 spin-half fields transforming as the 4 and 16 of $SO(5)_c$, respectively.

The bosonic Lagrangian is given as follows:

$$\begin{aligned}2\kappa^2 e^{-1}\mathcal{L}_{\text{boson}} = {} & R + \frac{1}{2}m^2(T^2 - 2T_{ij}T^{ij}) - \text{tr}(P_\mu P^\mu) \\ & - \frac{1}{2}(V^i_I V^j_J F^{IJ}_{\mu\nu})^2 + m^2(V_i^{-1I} C_{\mu\nu\rho I})^2 \\ & + e^{-1}\left(\frac{1}{2}\delta^{IJ}(C_3)_I \wedge (dC_3)_J \right. \\ & \left. + m\epsilon_{IJKLM}(C_3)_I F_2^{JK} F_2^{LM} + m^{-1}p_2(A,F)\right). \end{aligned} \tag{2.1}$$

Here $I, J = 1, 2, \ldots, 5$ denote $SO(5)_g$ indices, and $i, j = 1, 2, \ldots, 5$ denote $SO(5)_c$ indices. V^i_I represent 14 scalar degrees of freedom parametrizing the $SL(5,R)/SO(5)_c$ coset transforming as 5 under both $SO(5)_g$ and $SO(5)_c$. The tensor T_{ij} is given by $T_{ij} = V_i^{-1I} V_j^{-1J}\delta_{IJ}$ and $T = T_{ij}\delta^{ij}$. The covariant derivative of V^i_I is given as $\mathcal{D}_\mu V^i_I = \partial_\mu V^i_I - igA^a_\mu (J^a)_I^J V^i_J$, where $(J^a)_I^J$ are the generators in the vector representation of $SO(5)$. P_μ and Q_μ are symmetric and antisymmetric parts, respectively, of the covariant derivative: $V_i^{-1I}\mathcal{D}_\mu V^k_I \delta_{kj} = (Q_\mu)_{[ij]} + (P_\mu)_{(ij)}$.

In order to obtain a black hole solution, the theory can be simplified by truncating the non-Abelian theory with $SO(5)_g$ into $U(1)^2$, keeping only gauge fields along two of the Cartan generators [25,26]. The truncated theory has nonzero gauge fields along the generators J^{12} and J^{34}, denoted by $A^{(1)}_\mu$ and $A^{(2)}_\mu$, respectively, while all other components of the gauge fields as well as three-form potential $C^I_{\mu\nu\rho}$ are set to zero. $SO(5)_g$ can be identified with $SO(5)_c$ by making a gauge choice and restricting the vielbein to the form

$$V^i_I = \text{diag}(e^{-\phi_1} \quad e^{-\phi_1} \quad e^{-\phi_2} \quad e^{-\phi_2} \quad e^{2\phi_1+2\phi_2}). \tag{2.2}$$

With the above truncated field contents, the bosonic Lagrangian becomes [25]

$$\begin{aligned}2\kappa^2 e^{-1}\mathcal{L} = {} & R - \frac{1}{2}m^2\nu(\phi_1,\phi_2) - 6(\partial\phi_1)^2 - 6(\partial\phi_2)^2 \\ & - 8(\partial_\mu\phi_1)(\partial^\mu\phi_2) - e^{-4\phi_1}F^{(1)2}_{\mu\nu} \\ & - e^{-4\phi_2}F^{(2)2}_{\mu\nu} + m^{-1}p_2(A,F),\end{aligned}$$

$$\begin{aligned}\text{where } \nu(\phi_1,\phi_2) = {} & -8e^{2(\phi_1+\phi_2)} - 4e^{-2\phi_1-4\phi_2} - 4e^{-4\phi_1-2\phi_2} \\ & + e^{-8\phi_1-8\phi_2}. \end{aligned} \tag{2.3}$$

As shown in Refs. [25,26], the above truncated bosonic Lagrangian admits an asymptotically AdS black hole solution[1] with two charges given by the following metric:

$$\begin{aligned} & ds^2 = e^{2A(r)}(h(r)dt^2 - d\vec{x}^2) - \frac{e^{2B(r)}}{h(r)}dr^2, \\ & A^{(1)}_\mu dx^\mu = A^{(1)}(r)dt, \qquad A^{(2)}_\mu dx^\mu = A^{(2)}(r)dt, \\ & \phi_1 = \phi_1(r), \qquad \phi_2 = \phi_2(r), \end{aligned} \tag{2.4}$$

where the various functions depend only on r and are given by

$$\begin{aligned} e^{2A(r)} &= \frac{m^2}{4}r^2\left(1+\frac{Q_1^2}{r^4}\right)^{\frac{1}{5}}\left(1+\frac{Q_2^2}{r^4}\right)^{\frac{1}{5}}, \\ e^{2B(r)} &= \frac{4}{m^2}\frac{1}{r^2}\left(1+\frac{Q_1^2}{r^4}\right)^{-\frac{4}{5}}\left(1+\frac{Q_2^2}{r^4}\right)^{-\frac{4}{5}}, \\ h(r) &= \left(1-\frac{r^2}{r_h^2}\frac{(r_h^4+Q_1^2)(r_h^4+Q_2^2)}{(r^4+Q_1^2)(r^4+Q_2^2)}\right), \\ A^{(1)}(r) &= \frac{1}{2}\frac{m}{2}\frac{Q_1}{r_h}\left(\frac{r_h^4+Q_2^2}{r_h^4+Q_1^2}\right)^{\frac{1}{2}}\left(1-\frac{r_h^4+Q_1^2}{r^4+Q_1^2}\right), \\ A^{(2)}(r) &= \frac{1}{2}\frac{m}{2}\frac{Q_2}{r_h}\left(\frac{r_h^4+Q_1^2}{r_h^4+Q_2^2}\right)^{\frac{1}{2}}\left(1-\frac{r_h^4+Q_2^2}{r^4+Q_2^2}\right), \\ e^{2\phi_1(r)} &= r^{\frac{4}{5}}\frac{(r^4+Q_2^2)^{\frac{2}{5}}}{(r^4+Q_1^2)^{\frac{3}{5}}}, \qquad e^{2\phi_2(r)} = r^{\frac{4}{5}}\frac{(r^4+Q_1^2)^{\frac{2}{5}}}{(r^4+Q_2^2)^{\frac{3}{5}}}. \end{aligned} \tag{2.5}$$

[1]Reference [26] gave $k = 0, \pm 1$ solutions of the bosonic action, of which we consider $k = 0$.

$m/2$ is related to the radius of AdS L, by $m/2 = 1/L$ and $g = 2m$. The temperature and entropy density associated with this black hole are given, respectively, by [17]

$$T = \frac{m^2}{4}\frac{r_h}{2\pi}\frac{3 + \frac{Q_1^2}{r_h^4} + \frac{Q_2^2}{r_h^4} - \frac{Q_1^2 Q_2^2}{r_h^8}}{\sqrt{1 + \frac{Q_1^2}{r_h^4}}\sqrt{1 + \frac{Q_2^2}{r_h^4}}},$$
$$s = (m/2)^5 \frac{r_h}{4G}\sqrt{(r_h^4 + Q_1^2)(r_h^4 + Q_2^2)}. \qquad (2.6)$$

The chemical potentials and charge density are given by

$$\mu_1 = \frac{1}{2}\frac{m^2}{4}\frac{Q_1}{r_h}\left(\frac{r_h^4 + Q_2^2}{r_h^4 + Q_1^2}\right)^{\frac{1}{2}}, \qquad \mu_2 = \frac{1}{2}\frac{m^2}{4}\frac{Q_2}{r_h}\left(\frac{r_h^4 + Q_1^2}{r_h^4 + Q_2^2}\right)^{\frac{1}{2}},$$
$$\rho_i = \frac{Q_i s}{2\pi r_h^2}. \qquad (2.7)$$

As evident from the above expression, this two-charge black hole admits an extremal limit where $T = 0$:

$$3 + \frac{Q_1^2}{r_h^4} + \frac{Q_2^2}{r_h^4} - \frac{Q_1^2 Q_2^2}{r_h^8} = 0. \qquad (2.8)$$

As apparent from above, at zero temperature the entropy of this black hole solution does not vanish. In the following sections, we will consider fermion fluctuations in this background and investigate the possibility of the boundary theory admitting a Fermi surface.

III. FERMIONIC ACTION

The fermionic content of the $N = 4$ gauged supergravity in seven dimensions [22–24] consists of spin-3/2 gravitini ψ_μ^A and spin-1/2 field λ_i^A transforming under $SO(5)_c$ as **4** and **16**, respectively, where A and i are spinor and vector indices, respectively, of $SO(5)_c$ and they satisfy $\gamma^i\lambda_i = 0$.

We have used the notation of Ref. [23], but the metric is mostly negative. In addition, we introduce the following for convenience. We have chosen J^{12} and J^{34} as the two Cartan generators. For the vector representation, we rewrite the five components of a vector v^i as

$$v^{1\pm} = \frac{1}{\sqrt{2}}(v^1 \pm v^2), \qquad v^{2\pm} = \frac{1}{\sqrt{2}}(v^3 \pm v^4), \qquad v^0 = v^5, \qquad (3.1)$$

so that $U(1)\times U(1)$ charges associated are as follows: $v^{1\pm}$ and $v^{2\pm}$ have charges $(\mp 1, 0)$ and $(0, \mp 1)$, respectively, while ν^0 has charge (0,0).

Spinor representations of these two $SO(5)$ generators can be written by using $SO(5)$ γ matrices γ^i as

$$S^{12} = -(i/2)\gamma^1\gamma^2, \qquad S^{34} = -(i/2)\gamma^3\gamma^4. \qquad (3.2)$$

We have written the $SO(5)$ spinors as $\psi(s_{12}, s_{34})$, with s_{12}, $s_{34} = \pm\frac{1}{2}$ being the respective charges.

The terms in the Lagrangian consisting of only spin-1/2 fields λ^i are given by

$$e^{-1}\mathcal{L}_{\text{fermion}} = \frac{i}{2}\bar{\lambda}_i(\Gamma^\mu D_\mu \lambda^i) - \frac{m}{8}\bar{\lambda}_i(8T^{ij} - T\delta^{ij})\lambda_j - \frac{1}{32}\bar{\lambda}_i\gamma^j\gamma^{kl}\gamma^i\Gamma^{\mu\nu}\lambda_j(F_{\mu\nu})_{kl}, \qquad (3.3)$$

representing the kinetic term, mass term, and Pauli term. The covariant derivatives are given by

$$D_\mu\lambda^i = \nabla_\mu\lambda^i - ig[(A_\mu^{(1)}(J^{12})^i_j + A_\mu^{(2)}(J^{34})^i_j)\lambda^j + (A_\mu^{(1)}S^{12} + A_\mu^{(2)}S^{34})\lambda^i], \qquad (3.4)$$

where ∇_μ is the covariant derivative containing the spin connection and is given by

$$\nabla_\mu = \partial_\mu - \frac{1}{4}(\omega_\mu)_{ab}\Gamma^{ab} \qquad (3.5)$$

and J^{12}, J^{34} (S^{12}, S^{34}) are the vector (spinor) representations of the $U(1)\times U(1)$ generators.

The terms in the Lagrangian corresponding to coupling between gravitino ψ_μ and spin-1/2 fields λ^i are given by

$$e^{-1}\mathcal{L}_{\text{int}} = \bar{\psi}_\mu(-m\Gamma^\mu T_{ij}\gamma^i\lambda^j + \Gamma^\nu\Gamma^\mu(P_\nu)_{ij}\gamma^i\lambda^j + \frac{1}{2}\Gamma^{\nu\sigma}\Gamma^\mu(F_{\nu\sigma})_{ij}\gamma^i\lambda^j). \qquad (3.6)$$

As in Ref. [16], we would like to consider only those components of λ that do not couple to the gravitino. Since the charges of the gravitino under two Cartans of $SO(5)$ are $\pm\frac{1}{2}$, λ's having charges $\pm\frac{3}{2}$ under one generator or the other will not couple to the gravitino. A straightforward computation of (3.6) reveals that λ couples to the gravitino in the following four combinations: $\gamma^{1+}\lambda^{1-}$, $\gamma^{1-}\lambda^{1+}$, $\gamma^{2+}\lambda^{2-}$, and $\gamma^{2-}\lambda^{2+}$. Therefore, we will consider only the following eight components of λ, given in this notation by $\lambda^{1+}(-\frac{1}{2}, s_{34})$, $\lambda^{1-}(\frac{1}{2}, s_{34})$, $\lambda^{2+}(s_{12}, -\frac{1}{2})$, and $\lambda^{2-}(s_{12}, \frac{1}{2})$ with s_{12}, $s_{34} = \pm 1/2$, each of which has at least one of the charges equal to $\pm\frac{3}{2}$ in our notation.

So far as the dual field theory is concerned, according to the conjecture it is given by the six-dimensional (2, 0) conformal field theory. It has an R-symmetry group $SO(5)$, and the relevant field content is the tensor multiplet consisting of a self-dual 2-form potential $B_{\mu\nu}$ transforming as **1**, five scalars ϕ^i transforming as **5**, and four symplectic Majorana-Weyl spinors ψ^A transforming as **4** under the R-symmetry group. All the fields are taken to be in the adjoint representation of $U(N)$. As given above, $U(1)\times U(1)$ charges associated with various fields are as follows: $\phi^{1\pm}$ and $\phi^{2\pm}$ have charges $(\mp 1, 0)$ and $(0, \mp 1)$, respectively, while the charge of ϕ^0 is (0,0). Four spinors can be represented as $\psi(s_{12}, s_{34})$ with $s_{12}, s_{34} = \pm 1/2$. As explained in Refs. [27,28], operators dual to the spinors in the supergravity transforming under **16** are of the form $\text{tr}(\phi\psi)$. So, counting the $U(1)\times U(1)$ charges, the pertinent operators may be organized as $\mathrm{tr}(\phi^{1\pm}\psi(s_{12}, s_{34}))$ and $\mathrm{tr}(\phi^{2\pm}\psi(s_{12}, s_{34}))$ with s_{12}, $s_{34} = \pm 1/2$, as given in Table I.

IV. DIRAC EQUATIONS

In order to study Fermi surfaces in the boundary theory, we will consider solutions of the Dirac equations for the eight spinors that do not couple to the gravitino. The Dirac equations that follow from the fermionic Lagrangian (3.3), after setting the components of fermions that couple to the gravitino equal to zero, give rise to the following equations:

$$\begin{aligned}[i\Gamma^\mu\nabla_\mu - m(m_1 e^{2\phi_1} + m_2 e^{2\phi_2} + m_3 e^{-4(\phi_1+\phi_2)}) \\ + 2m(q_1 A_\mu^{(1)} + q_2 A_\mu^{(2)}) \\ + i(p_1 e^{-2\phi_1} F_{\mu\nu}^{(1)} + p_2 e^{-2\phi_2} F_{\mu\nu}^{(2)})\Gamma^{\mu\nu}]\lambda^i(s_{12}, s_{34}) = 0,\end{aligned} \tag{4.1}$$

where we introduce parameters m_1, m_2, m_3, q_1, q_2, p_1, and p_2. The values of the parameters for different spinors are summarized in Table I.

One can observe the top four rows are interchanged with the bottom four rows under $(m_1, q_1, p_1) \leftrightarrow (m_2, q_2, p_2)$, and so the Dirac equations remain unchanged provided one interchanges the charge parameters Q_1 and Q_2 as well. Under $q_i \to -q_i, p_i \to -p_i$, (1,4), (2,3), (5,8), and (6,7) will be interchanged between each other.

We consider the Dirac Eqs. (4.1) in the bosonic background (2.4), (2.5). Our choices of Γ matrices, with tangent space indices, are as follows:

$$\begin{aligned}\Gamma^{\hat{t}} &= \begin{pmatrix} \Gamma_1^{\hat{t}} & 0 \\ 0 & \Gamma_1^{\hat{t}} \end{pmatrix}, \quad \Gamma^{\hat{r}} = \begin{pmatrix} \Gamma_1^{\hat{r}} & 0 \\ 0 & \Gamma_1^{\hat{r}} \end{pmatrix}, \quad \Gamma^{\hat{i}} = \begin{pmatrix} \Gamma_1^{\hat{i}} & 0 \\ 0 & \Gamma_1^{\hat{i}} \end{pmatrix}, \\ \Gamma_1^{\hat{t}} &= \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_1 \end{pmatrix}, \quad \Gamma_1^{\hat{r}} = \begin{pmatrix} i\sigma_3 & 0 \\ 0 & i\sigma_3 \end{pmatrix}, \quad \Gamma_1^{\hat{i}} = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix},\end{aligned} \tag{4.2}$$

where σ_1, σ_2, and σ_3 are Pauli spin matrices.

The effect of spin connection in the covariant derivative $\nabla_\mu\lambda^i = (\partial_\mu - \frac{1}{4}(\omega_\mu)_{ab}\Gamma^{ab})\lambda^i$ may be canceled by introducing

$$\lambda^i = e^{-A(r)}h(r)^{-1/4}\chi^i. \tag{4.3}$$

Apart from r dependence, we choose the time and space dependence as

$$\chi^i(t, r, x) = e^{-i\omega t + ikx}\chi^i(r). \tag{4.4}$$

We can write χ^i as consisting of two four-component spinors, Ψ^i and η^i, as follows:

$$\chi^i = \begin{pmatrix} \Psi^i \\ \eta^i \end{pmatrix}. \tag{4.5}$$

From the block diagonal structure of the Γ matrices as given in first line of (4.2), it is clear that both Ψ^i and η^i will satisfy the same equation, and so it is sufficient to study the equations satisfied by Ψ^i. We have chosen

$$\Psi^i = \begin{pmatrix} \Psi_1^i \\ \Psi_2^i \end{pmatrix}, \qquad \Psi_\alpha^i = \begin{pmatrix} \Psi_{\alpha-}^i \\ \Psi_{\alpha+}^i \end{pmatrix}, \qquad \alpha = 1, 2. \tag{4.6}$$

With these notations, the Dirac equations in the bosonic background (2.4), (2.5) reduce to

$$\begin{aligned}\left(\partial_r + m\frac{e^B}{\sqrt{h}}M(\phi_1, \phi_2)\right)\Psi_{\alpha-} \\ = \frac{e^{B-A}}{\sqrt{h}}[u(r) + (-1)^\alpha k - v(r)]\Psi_{\alpha+}, \\ \left(\partial_r - m\frac{e^B}{\sqrt{h}}M(\phi_1, \phi_2)\right)\Psi_{\alpha+} \\ = \frac{e^{B-A}}{\sqrt{h}}[-u(r) + (-1)^\alpha k - v(r)]\Psi_{\alpha-},\end{aligned} \tag{4.7}$$

where $\alpha = 1$, 2 and we have introduced other functions given as follows:

TABLE I. Parameters and dual operators corresponding to various fermionic modes

No.	$\lambda^I(s_{12}, s_{34})$	m_1	m_2	m_3	q_1	q_2	p_1	p_2	Dual operator
1	$\lambda^{1-}(\frac{1}{2}, \frac{1}{2})$	$\frac{3}{2}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\mathrm{tr}(\phi^{1-}\psi(\frac{1}{2}, \frac{1}{2}))$
2	$\lambda^{1-}(\frac{1}{2}, -\frac{1}{2})$	$\frac{3}{2}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{3}{2}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\mathrm{tr}(\phi^{1-}\psi(\frac{1}{2}, -\frac{1}{2}))$
3	$\lambda^{1+}(-\frac{1}{2}, \frac{1}{2})$	$\frac{3}{2}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\mathrm{tr}(\phi^{1+}\psi(-\frac{1}{2}, \frac{1}{2}))$
4	$\lambda^{1+}(-\frac{1}{2}, -\frac{1}{2})$	$\frac{3}{2}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{3}{2}$	$-\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\mathrm{tr}(\phi^{1+}\psi(-\frac{1}{2}, -\frac{1}{2}))$
5	$\lambda^{2-}(\frac{1}{2}, \frac{1}{2})$	$-\frac{1}{2}$	$\frac{3}{2}$	$-\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\mathrm{tr}(\phi^{2-}\psi(\frac{1}{2}, \frac{1}{2}))$
6	$\lambda^{2-}(-\frac{1}{2}, \frac{1}{2})$	$-\frac{1}{2}$	$\frac{3}{2}$	$-\frac{1}{4}$	$-\frac{1}{2}$	$\frac{3}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\mathrm{tr}(\phi^{2-}\psi(-\frac{1}{2}, \frac{1}{2}))$
7	$\lambda^{2+}(\frac{1}{2}, -\frac{1}{2})$	$-\frac{1}{2}$	$\frac{3}{2}$	$-\frac{1}{4}$	$\frac{1}{2}$	$-\frac{3}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\mathrm{tr}(\phi^{2+}\psi(\frac{1}{2}, -\frac{1}{2}))$
8	$\lambda^{2+}(-\frac{1}{2}, -\frac{1}{2})$	$-\frac{1}{2}$	$\frac{3}{2}$	$-\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{3}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\mathrm{tr}(\phi^{2+}\psi(-\frac{1}{2}, -\frac{1}{2}))$

$$M(\phi_1,\phi_2) = (m_1 e^{2\phi_1} + m_2 e^{2\phi_2} + m_3 e^{-4(\phi_1+\phi_2)}),$$
$$u(r) = \frac{1}{\sqrt{h}}[\omega + 2m(q_1 A_t^{(1)} + q_2 A_t^{(2)})],$$
$$v(r) = 2e^{-B}[p_1 e^{-2\phi_1} F_{rt}^{(1)} + p_2 e^{-2\phi_2} F_{rt}^{(2)}]. \quad (4.8)$$

These two first-order coupled equations can be written as two decoupled equations of second order:

$$\partial_r^2 \Psi_{\alpha\pm} - \partial_r \log\left(\frac{e^{B-A}}{\sqrt{h}}[v-(-1)^\alpha k \pm u]\right)\partial_r \Psi_{\alpha\pm} + \left[\mp\partial_r\left(\frac{e^B}{\sqrt{h}} mM\right) - \frac{m^2 M^2 e^{2B}}{h} + \frac{e^{2(B-A)}}{h}[u^2 - (v-(-1)^\alpha k)^2] \pm \frac{e^B}{\sqrt{h}} mM \partial_r \log\left(\frac{e^{B-A}}{\sqrt{h}}[v-(-1)^\alpha k \pm u]\right)\right]\Psi_{\alpha\pm} = 0. \quad (4.9)$$

$\alpha = 1, 2$ are related through flipping the sign of k. Equations (4.9) are invariant under flipping the sign of q_i, p_i, ω, and k. Therefore, the spinors and their conjugates, which are related to each other through flipping of the signs of q_i, p_i, will have solutions related through a change of the signs of k and ω.

A. Asymptotic limit

We consider the asymptotic limit of (4.9) following Ref. [16]. For $r \to \infty$, $e^{2\phi_1} = e^{2\phi_2} = 1$, implying $M(\phi_1, \phi_2) = (m_1 + m_2 + m_3)$. Substituting this and limiting values for other expressions, we obtain

$$\partial_r^2 \Psi_{\alpha\pm} + \frac{2}{r}\partial_r \Psi_{\alpha\pm} - \frac{(2M_0)^2 \pm 2M_0}{r^2}\Psi_{\alpha\pm} = 0, \quad (4.10)$$

where we use $M_0 = m_1 + m_2 + m_3$, giving rise to the following solutions:

$$\Psi_{\alpha+} = A_\alpha(k) r^{2M_0} + B_\alpha(k) r^{-2M_0-1},$$
$$\Psi_{\alpha-} = C_\alpha(k) r^{2M_0-1} + D_\alpha(k) r^{-2M_0}. \quad (4.11)$$

A relation between $A_\alpha(k)$ $[B_\alpha(k)]$ and $C_\alpha(k)$ $[D_\alpha(k)]$ can be obtained [16,17,29] from the first-order Dirac Eqs. (4.7):

$$C_\alpha = \frac{\tilde{\omega} + (-1)^\alpha k}{2(2M_0) - 1} A_\alpha, \qquad B_\alpha = \frac{\tilde{\omega} - (-1)^\alpha k}{2(2M_0) + 1} D_\alpha,$$

where $\tilde{\omega} = \omega + 4\sum_{i=1}^{2} q_i A_t^{(i)}(r \to \infty)$. (4.12)

For $M_0 > 0$, A is the source term and D is the response. One can obtain the retarded Green function for the dual fermionic operator as

$$D_\alpha = (G_R)_{\alpha\beta} A_\beta, \quad (4.13)$$

where the fermionic fluctuation satisfy the infalling boundary condition at the horizon. In this case, differential Eqs. (4.7) do not mix different α components, rendering the Green function to be diagonal.

B. Near horizon limit

In order to find the infalling boundary condition, following Ref. [16], we consider a near horizon analysis of the Dirac Eqs. (4.9) in the extremal case. For the extremal case, ($T = 0$) metric develops a double pole at the horizon. We expand the metric and other bosonic fields near the horizon. The terms up to leading order are given in the following. We have also introduced [7,16] parameters useful to analyze the Dirac equation in the near horizon region:

$$g_{ii} \sim k_0^2 = \frac{m^2}{4} r_h^{-8/5} 2^{3/5} \frac{(Q_1^2 + r_h^4)^{4/5}}{(Q_1^2 - r_h^4)^{2/5}},$$
$$g_{tt} \sim \tau_0^2 = \frac{m^2}{4} r_h^{-4/5} 2^{6/5} \frac{(Q_1^4 + 6Q_1^2 r_h^4 - 3r_h^8)}{(Q_1^2 + r_h^4)^{8/5}(Q_1^2 - r_h^4)^{1/5}},$$
$$g_{rr} \sim L_2^2 = \frac{4}{m^2} 2^{-9/5} r_h^{16/5} \frac{(Q_1^2 + r_h^4)^{2/5}(Q_1^2 - r_h^4)^{4/5}}{(Q_1^4 + 6Q_1^2 r_h^4 - 3r_h^8)},$$
$$A_t^{(i)} \sim \beta_i (r - r_h), \qquad \beta_1 = \frac{m}{2}\frac{4\sqrt{2} r_h^4 Q_1}{(Q_1^2 + r_h^4)\sqrt{(Q_1^2 - r_h^4)}},$$
$$\beta_2 = \frac{m}{2}\frac{2}{\sqrt{2} r_h^2}\frac{(Q_1^2 + 3r_h^4)^{1/2}(Q_1^2 - r_h^4)}{(Q_1^2 + r_h^4)},$$
$$e^{2\phi_1} \sim e^{2\phi_{10}} = \frac{2^{2/5} r_h^{12/5}}{(Q_1^2 - r_h^4)^{2/5}(Q_1^2 + r_h^4)^{1/5}},$$
$$e^{2\phi_2} \sim e^{2\phi_{20}} = \frac{1}{2^{3/5} r_h^{8/5}}\frac{(Q_1^2 - r_h^4)^{3/5}}{(Q_1^2 + r_h^4)^{1/5}}. \quad (4.14)$$

At the near horizon limit, the Dirac equation simplifies into the following [16]:

$$\partial_r^2 \Psi_{\alpha\pm} + \frac{1}{r - r_h}\partial_r \Psi_{\alpha\pm} - \frac{\nu_k^2}{(r - r_h)^2}\Psi_{\alpha\pm} = 0,$$

where $\nu_k^2 = (mM(\phi_{10}, \phi_{20}))^2 L_2^2 + (\tilde{k}/k_0))^2 L_2^2 - (2m)^2 (q_1 e_1 + q_2 e_2)^2$

and $\tilde{k} = k - (-1)^\alpha \frac{2k_0}{L_2^2}[p_1 e^{-2\phi_{10}} e_1 + p_2 e^{-2\phi_{20}} e_2]$,

$$e_i = (L_2/\tau_0)\beta_i, \quad (4.15)$$

where the expressions for various parameters are given in (4.14).

We need to analyze solutions to (4.15) following Refs. [7,16] to determine the appropriate boundary condition. It admits two solutions:

$$\Psi \sim (r - r_h)^{\pm\nu_k}, \tag{4.16}$$

of which we choose

$$\Psi \sim (r - r_h)^{+\nu_k}, \tag{4.17}$$

which corresponds to the infalling boundary condition at the horizon. With this boundary condition, we look for Fermi momentum $k = k_F$ for which the source term $A_\alpha(k_F) = 0$ in (4.11).

As evident from the second equation of (4.15), the square of the exponent ν_k^2 has positive contributions from the mass and shifted momentum and a negative contribution from coupling to electric fields. When the electric field is strong enough, ν_k will become imaginary for a range of values of momentum [16] which is called the oscillatory region. Over this range, the Green function becomes oscillatory with oscillatory peaks periodic in $\log|\omega|$ [6,7]. For scalar particles, this corresponds to an instability towards pair production in AdS_2 region. For spinors, however, it does not imply an instability, as there is no singularity of the Green function in the upper half of the complex ω plane [7]. On the field theory side, the oscillatory region implies that the effective dimension of the operator in the boundary conformal field theory will become complex [7].

The boundary of the oscillatory region is given by $\tilde{k} = \tilde{k}_{osc}$, where

$$\tilde{k}^2_{\rm osc} = k_0^2\left[\left(\frac{g}{L_2}\right)^2(q_1e_1 + q_2e_2)^2 - m^2M_0^2\right], \tag{4.18}$$

and so the oscillatory region depends on the relative strengths of charges and the masses.

The retarded Green function near the Fermi surface for small ω can be written in terms of $k_\perp = k - k_F$ and ω as follows [7,16]:

$$G_R(k,\omega) \sim \frac{h_1}{k_\perp - \omega/v_F - h_2 e^{i\gamma_{k_F}}(2\omega)^{2\nu_{k_F}}},$$
$$\gamma_k = \arg[\Gamma(-2\nu_k)(e^{-2\pi i\nu_k} - e^{-2\pi g(q_1e_1+q_2e_2)})], \tag{4.19}$$

where ν_k is given in (4.15), γ_k is as given above, and h_1 and h_2 are positive constants. For $\nu_{k_F} > 1/2$, the leading real part comes from $\mathcal{O}(\omega)$ correction, given by the $1/v_F$ term. The ratio of excitation width to excitation energy $\frac{\Gamma}{\omega_*}$ becomes vanishing as the Fermi surface is approached. Quasiparticles are stable, and the system behaves as a Fermi liquid.

For $\nu_{k_F} < 1/2$, one ignores the v_F term, and the dispersion relation becomes

$$\omega_* \sim (k_\perp)^z, \quad \text{where } z = \frac{1}{2\nu_{k_F}}, \tag{4.20}$$

and the ratio of excitation width to energy is

$$\frac{\Gamma}{\omega_*} = \begin{cases} \tan\left(\frac{\gamma_{k_F}}{2\nu_{k_F}}\right) & k_\perp > 0, \\ \tan\left(\frac{\gamma_{k_F}}{2\nu_{k_F}} - \pi z\right) & k_\perp < 0. \end{cases} \tag{4.21}$$

For $\nu_{k_F} = 1/2$, is the marginal Fermi liquid, where $\frac{\Gamma}{\omega_*}$ vanishes logarithmically in ω as one approaches the Fermi surface. We find Fermi surfaces for our cases corresponding to $\nu_{k_F} > 1/2$, $\nu_{k_F} = 1/2$, and $\nu_{k_F} < 1/2$.

V. FERMI SURFACES

In order to study Fermi momentum, we have considered Dirac Eq. (4.15) with infalling boundary condition (4.17). We numerically solve it for different values of k to find k_F where $A_\alpha(k_F)$ given in (4.11) vanishes. We considered a lower sign of Ψ and $\alpha = 1$ in (4.15), as other components will give similar results.

As is clear from Table I, under the interchange of two Cartan generators, the top four rows are interchanged with the bottom four rows, and so interchanging charge parameters $Q_1 \leftrightarrow Q_2$ we will get the same equations. Therefore, it is sufficient to consider only the four top modes. Moreover, flipping the sign of q_i and p_i will interchange spinors with its conjugates. As we have already mentioned, from the symmetry of Dirac Eq. (4.9), that amounts to flipping the sign of k and ω. So if a fermion with (q_i, p_i) contains a Fermi surface singularity at k_F, its conjugate one with $(-q_i, -p_i)$ will have a Fermi surface singularity at $-k_F$. Under flipping the signs of q_i and p_i rows (1, 2) will be interchanged with rows (3, 4) in Table I. Therefore, in the present case, we have considered only rows 3 and 4 of Table I. We numerically searched for a Fermi momentum for these two spinors for $\omega = 0$ and have plotted the k vs the square of the inverse charge parameter, given by $x = r_h^4/Q_1^2$. The range of x is restricted to (0,1), as $Q_1^2/r_h^4 < 1$ at $T = 0$ will make Q_2^2/r_h^4 negative. At $x = 1/3$, $Q_1 = Q_2$ and two chemical potentials will be equal to each other. We introduce $\tilde{Q}_1 = Q_1/r_h^2$, which is the value of the charge parameter in units of r_h^2. It is related to x through $x = 1/\tilde{Q}_1^2$. For simplicity, $m/2$ is set to be equal to 1 in the numerical computation. The discussions of each of these cases are given below.

Case 1: $\lambda^{1+}(-\frac{1}{2}, -\frac{1}{2})$, $q_1 = -\frac{3}{2}$, $q_2 = -\frac{1}{2}$; *dual operator* $\mathrm{tr}(\phi^{1+}\psi(-\frac{1}{2}, -\frac{1}{2}))$.—This case corresponds to higher net charge $|q_1 + q_2| = 2$ and is more likely to have a Fermi surface. Indeed, we find two branches of Fermi surface singularities as shown in Fig. 1. We find that one of them corresponds to non-Fermi liquid, while the other one is in

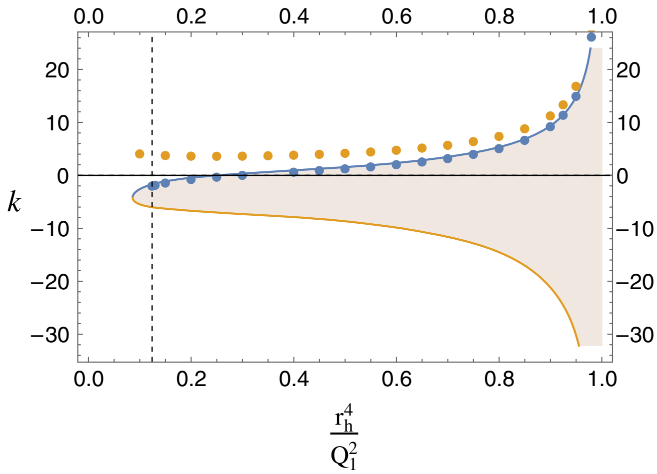

FIG. 1. Case 1: k vs r_h^4/Q_1^2. The shaded bell-shaped area represents the oscillatory region. It shows two branches; the dashed line shows where the lower branch enters the oscillatory region.

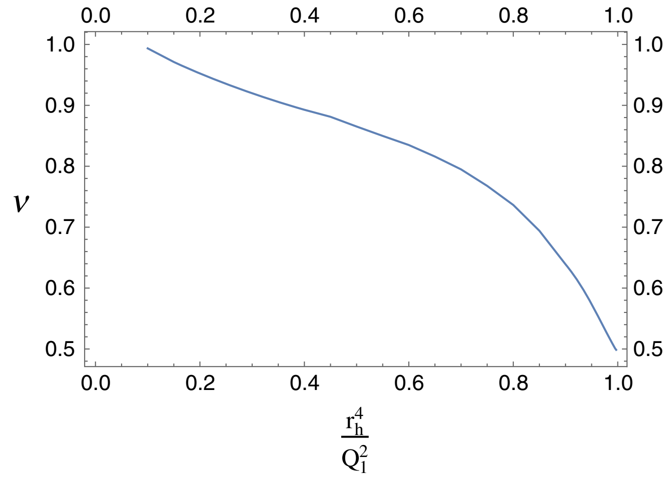

FIG. 3. Case 1: ν vs r_h^4/Q_1^2 for the upper branch.

the regime of Fermi liquid. Starting from the left, as $\tilde{Q}_1$ decreases from ∞ to 3.356, there is no oscillatory region. It first appears at $\tilde{Q}_1 = 3.356$ and expands as the charge parameter decreases till $\tilde{Q}_1 = 1$, carving out a bell-shaped region.

The branch for the Fermi surface in the non-Fermi region appears along the boundary of the oscillatory region. It starts at $\tilde{Q}_1 = 2.887$, on the left of which it disappears inside the oscillatory region, implying the operator develops a complex dimension. As $\tilde{Q}_1$ decreases, we find the Fermi momenta to be very close to the boundary of the oscillatory region, almost trace the boundary itself, and continue till $\tilde{Q}_1 = 1$. Looking at ν and Γ/ω for this branch (Fig. 2), we observe that ν is small where it vanishes inside the oscillatory region on the left, increases as $\tilde{Q}_1$ decreases, and approaches 0.5 as $\tilde{Q}_1 = 1$. Therefore, the Fermi surface remains in the non-Fermi region for the entire range, while it approaches marginal Fermi-liquid behavior at $\tilde{Q}_1 = 1$, which corresponds to $Q_2^2/r_h^4 \to \infty$. The excitation width decreases steadily with $\tilde{Q}_1$, implying the quasiparticle will become more and more stable and approaches zero at the limit of marginal Fermi liquid, where $\tilde{Q}_1$ tends to 1. This branch passes through zero at around $\tilde{Q}_1 = 2$, which may be associated with a transition between Fermi surfaces of particles to antiparticles [16].

The other branch appears on the same side of the oscillatory region, and momenta are at a larger value. It is extended over the entire range and does not enter the oscillatory region. Plots of ν are given in Fig. 3, and, as one may observe, ν decreases as the charge parameter decreases. On the left extreme, where $\tilde{Q}_1$ approaches ∞, ν approaches 1, while at the other extreme where $\tilde{Q}_1$ approaches 1, ν approaches 0.5, indicating the limiting behavior to be that of marginal Fermi liquid. Throughout the range, it remains in the Fermi-liquid regime and the limit of $\tilde{Q}_1 = 1$ becomes marginal.

Considering both the branches together, one may observe, over a range of $\tilde{Q}_1$ in Fig. 1, that there are two Fermi surfaces. The signs of the Fermi momenta of these

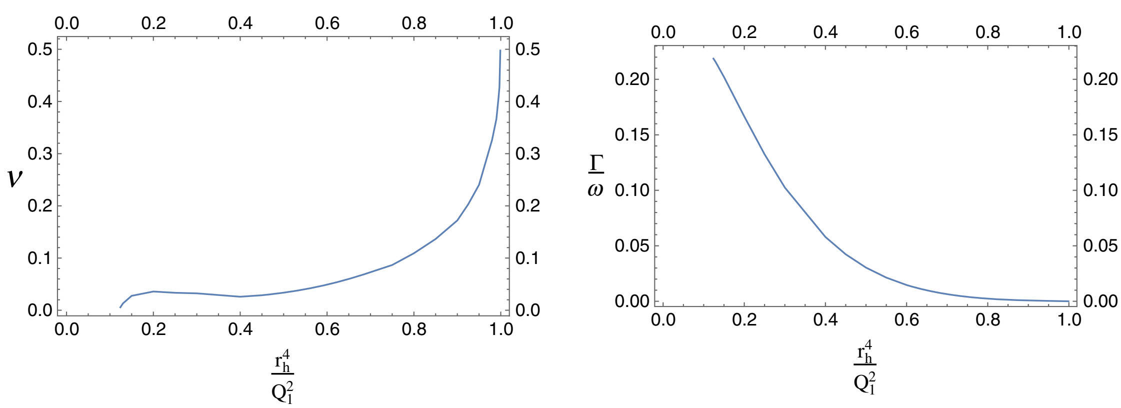

FIG. 2. Case 1: ν vs r_h^4/Q_1^2 and Γ/ω vs r_h^4/Q_1^2 for the lower branch.

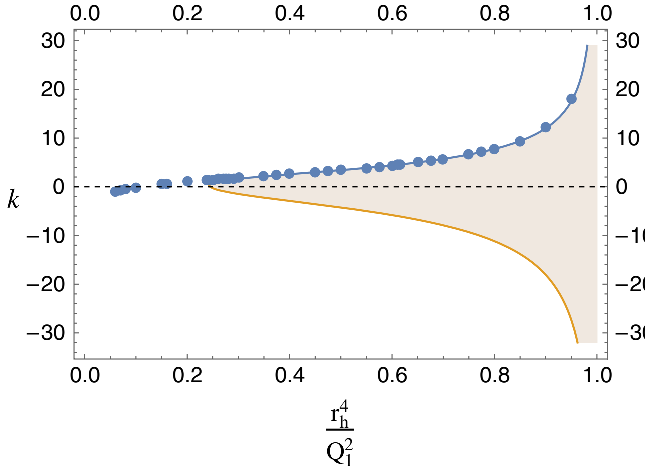

FIG. 4. Case 2: k vs r_h^4/Q_1^2. The shaded bell-shaped area represents the oscillatory region.

two surfaces are the same for $\tilde{Q}_1 < 2$ and opposite for $\tilde{Q}_1 > 2$. For this range of charge parameter, the dual field theory admits two different excitations, which give rise to two different Fermi surfaces. As $\tilde{Q}_1$ approaches 1, branches approach each other and merge, giving rise to marginal Fermi liquid in the limiting case.

Case 2: $\lambda^{1+}(-\frac{1}{2},\frac{1}{2})$ *and* $q_1 = \frac{1}{2}, q_2 = -\frac{3}{2}$; *dual operator* $\mathrm{tr}(\phi^{1+}\psi(-\frac{1}{2},\frac{1}{2}))$.—This case corresponds to a lower total charge compared to that of case 1 (total charge $|q_1 + q_2| = 1$), and we have found one branch of the Fermi surface as shown in Fig. 4. Here also there is no oscillatory region on the left side corresponding to a large value of the charge parameter. As $\tilde{Q}_1$ decreases from ∞, the oscillatory region appears at around $\tilde{Q}_1 = 2.04$ and expands till $\tilde{Q}_1 = 1$. In this case, the branch appears over the entire range and does not enter the oscillatory region. Plots for ν and Γ/ω for this branch are given in Fig. 5. On the left extreme, where the charge parameter approaches ∞, ν is greater than 0.5, indicating the Fermi-liquid regime. As $\tilde{Q}_1$ decreases, ν decreases steadily, and at $\tilde{Q}_1 = 4.23$, ν becomes equal to 0.5, which corresponds to marginal Fermi liquid. As $\tilde{Q}_1$ decreases further, ν reaches a minimum and then starts increasing with decreasing $\tilde{Q}_1$, reaching 0.5 again at the right extreme where $\tilde{Q}_1 = 1$. The excitation width decreases with decreasing $\tilde{Q}_1$, and at $\tilde{Q}_1 = 1$ it becomes very small, making the excitation stable. However, with increasing $\tilde{Q}_1$ it shoots up before it reaches the marginal Fermi-liquid state. For most of the range, the sign of the Fermi momentum remains positive, while it passes through zero at $\tilde{Q}_1 = 3.01$ and becomes negative at higher values, indicating a transition between Fermi surfaces of particles and antiparticles. This branch exhibits a transition from non-Fermi-liquid to Fermi-liquid behavior, with variations of the charge parameter. A similar transition has been observed in Ref. [30] in a different context, where it changes from Fermi liquid to non-Fermi liquid over variations of some impurity parameter.

To summarize, we have studied Fermi surfaces in the six-dimensional (2,0) theory using holographic correspondence. We consider an asymptotically AdS black hole solution in seven-dimensional gauged supergravity and analyze fermionic modes that do not couple to the gravitino. We considered two types of fermionic modes; with net charge $q = |q_1 + q_2| = 1$, 2, where q_1 and q_2 are the charges with respect to the two $U(1)$ gauge fields. We find that each fermionic mode admits a Fermi surface(s). In particular, fermions with $q = 2$ admit two branches of Fermi surfaces, of which one remains in the Fermi-liquid regime, while the other in the non-Fermi regime. For the other fermionic mode, we have found that the Fermi surface is in the regime of Fermi liquid for a range of charge parameters, while it is in the non-Fermi regime for the rest.

In the present model, we find one Fermi surface that is in the Fermi-liquid regime. Compared to other models with maximal supersymmetry, this is new in the present model, as such behaviors were not found in the case of $N = 4$ SYM and ABJM theory. The other Fermi surface remains in the deep non-Fermi regime and approaches marginal Fermi liquid for the limiting value of the charge parameter, which is very similar to the cases obtained in those models.

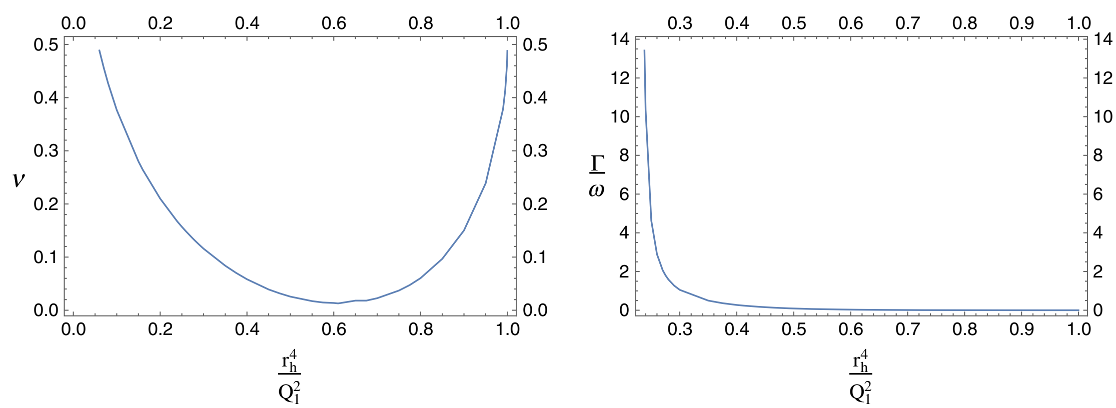

FIG. 5. Case 2: ν vs r_h^4/Q_1^2 and Γ/ω vs r_h^4/Q_1^2.

We have also found a branch which changes from Fermi liquid to non-Fermi liquid, which is different from $N = 4$ SYM and ABJM theory. For two of our branches, the Fermi momenta occur along the boundary of the oscillatory region, which is again very similar to results obtained in Refs. [16,17]. Among all the fermions that we have considered, it turns out that the nature of the Fermi surface depends on the $U(1) \times U(1)$ charge of fermions in the boundary theory and does not depend on the scalars. This behavior is different from $N = 4$ SYM, where the Fermi surface depends on the scalars in the boundary theory as well.

In the present work, we have considered Fermi momentum in the extremal limit at $\omega = 0$. A full-fledged study of spectral functions and transport phenomena at a finite temperature and over a range of ω may provide a more elaborate picture of the field theory. We have observed fermionic modes show marginal Fermi-liquid behavior for certain values of the charge parameter, for example, in the limit of $Q_1/r_h^2 = 1$, which at zero temperature corresponds to Q_2/r_h^2 approaching ∞. Marginal Fermi liquid was proposed to describe optically doped cuprates [31], and it may be interesting to further explore the model for these values of the charge parameter. The present model with a single charge by setting the other charge density to zero would also be a natural extension, which cannot be obtained as a limiting case of this study. Considering the quasiparticles on the field theory side, operators are given by $\mathrm{Tr}(\phi\psi)$ in our case, and we have observed that the nature of the Fermi surface changes only when the charge of the gaugino (ψ) in the operator changes, while it remains the same with the change of the scalar (ϕ). This is consistent with the argument [15] that the gaugino itself generates the Fermi surface. However, in $N = 4$ SYM, it was found that the nature of the Fermi surface may depend on bosons as well, and so this issue requires a better understanding.

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